

Chiral symmetry breaking in confining theories and asymptotic limits of operator product expansion

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ABSTRACT: The pattern of spontaneous chiral symmetry breaking (CSB) in confining background fields is analyzed. It is explicitly demonstrated how to get the inverse square root large proper time asymptotic of the operator product expansion which is needed for CSB.

KEYWORDS: Confinement, QCD, Spontaneous Symmetry Breaking, Phenomenological Models.

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1. Introduction

The phenomenon of spontaneous chiral symmetry breaking (CSB) is one of the two most important nonperturbative properties of quantum chromodynamics (QCD). Another property, confinement of color, is believed to be deeply connected with CSB. It is interesting to see how confinement and CSB sometimes play competitive roles when one speaks about low-energy hadron physics. Namely, one can find statements in the literature that confinement is “not seen” in low-energy spectroscopy, and it is possible to model the QCD vacuum by a set of classical field configurations (as one successfully does, for example, in the instanton model [1], see reviews [2, 3] and references therein), despite the fact that such ensembles typically have no confinement in the sense of area law of the Wilson loop. There is also an alternative line of arguments, going back to the good old constituent models, which states the “supremacy” of confinement (understood in terms of the confining string formation and corresponding linear potential between quarks), while CSB is to be derived from confinement (see, e.g. [4]) and there is no need in instantons or alike in this approach. In fact, these two points of view are to some extent complementary to each other, as the example of pion clearly shows: pion is simultaneously Nambu-Goldstone boson which should be massless in the chiral limit and, on the other hand, it is a bound state of quarks and antiquarks, as any other meson in QCD is.

The two physical pictures outlined above correspond to two different approaches one usually uses analyzing the phenomenon of CSB in QCD. The first approach concentrates on studies of the Dirac operator zero modes in this or that gauge field background. The chiral condensate is related in this case to the (quasi)-zero modes density by the well known Banks-Casher relation [5]. The model dependence enters when one starts to answer

an important question about physical relevance of the chosen background, in particular, about its confining properties. We refer the interested reader to the review [2] and references therein for the details of this approach.

Alternatively, one can choose this or that interaction kernel (for example, manifestly providing confinement) and construct gap equation for the chiral condensate with this kernel (see, e.g. recent paper [6]). The same procedure is commonly used in finite density QCD [7]. This exhibits in a very clear (but model-dependent) way the general relation between confinement and CSB [8], taking confinement as *the cause* of CSB.

In Shifman-Vainshtein-Zakharov sum rules approach [9] the chiral condensate $\langle \bar{q}q \rangle$ enters as an input parameter not directly related to, for example, gluon condensate $\langle \alpha_s F^2 \rangle$. In other words, there is no simple relation like $\langle \bar{q}q \rangle = \text{const} \cdot \langle \alpha_s F^2 \rangle^{3/4}$ one might naively think of.¹ In fact, studying correlators of hadronic currents one can get more sophisticated relations (see, e.g. [10]) between chiral and gluon condensates (and other nonperturbative quantities like f_π). The problem however is that SVZ approach misses the relation between chiral symmetry breaking and confinement. For example, nonzero value of $\langle \alpha_s F^2 \rangle$ does not at all indicate that the vacuum confines.² As is well known, one possible way to take the effects of confinement into account properly is to consider the dynamics of nonlocal objects like the Wilson loops.

In the present paper we address the problem of CSB in the first-quantized language, i.e. in terms of quark trajectories and not fields. In this sense, we are closer to the latter approach discussed above and not to the former one. The criterium for CSB in this framework is given by well known Banks-Casher asymptotic law (eq. (2.9) of this paper). We are going to address the following question: how this asymptotic law follows from the properties of the Wilson loop expansion over local condensates and their derivatives. Speaking differently, we are looking for the simplest subseries of the Wilson loop operator product expansion, whose summation provides CSB at the level of one-loop effective action. It is shown that CSB has to do with the large proper time asymptotic limit of the specific confining nonlocal gauge-invariant correlator of gluon fields.

2. One-loop effective action

We start from the expression for the Euclidean QCD partition function:

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu^a \mathcal{D}q \mathcal{D}q^\dagger \exp \left(-\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a + \int d^4x q^\dagger (i\gamma_\mu D_\mu - im) q \right) \\ &= \langle \det (i\gamma_\mu D_\mu - im) \rangle \end{aligned} \tag{2.1}$$

and we confine our attention to the case when the mass matrix is proportional to the unit matrix in flavor space with the eigenvalue m . As usual, the average of any gauge-invariant

¹It is worth reminding that nonperturbative gluon condensate is not a local order parameter, while chiral condensate is.

²As one can see on the lattice at large temperatures [11].

operator $\mathcal{R}(A)$ over gauge fields is understood with the standard Yang-Mills action

$$\langle \mathcal{R}(A) \rangle = \int \mathcal{D}A_\mu^a \mathcal{R}(A) \exp\left(-\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F_{\mu\nu}^a\right) \quad (2.2)$$

where normalization factor, proper gauge-fixing and ghost terms are included in the integration measure. The chiral condensate is given by the standard form

$$\langle \bar{q}q \rangle^{(M)} = i \langle q^\dagger q \rangle^{(E)} = -\frac{1}{V} \frac{\partial \log Z}{\partial m} = -\frac{\partial \Gamma_{\text{eff}}}{\partial m} \quad (2.3)$$

and the superscripts M, E stays for Minkowski and Euclidean values. It is assumed that the right hand side of (2.3) does not vanish in the limit $m \rightarrow 0$. This nonzero condensate corresponds to spontaneous CSB.

We are going to exploit the so-called Feynman-Schwinger representation technique, whose essence goes back to the seminal papers [12, 13]. There are basically two lines of use of this approach in the modern research. The first one exploits advantages the path integration provides for gauge-invariant formulation of relativistic bound state problems (see review [14] and references therein). The second line [15, 16] concentrates on the loop calculations in perturbative field theory (for review see [17]), which is sometimes simpler in world-line approach than in conventional Feynman diagrammatic framework. We collect some relevant formulas in appendix A for convenience, while the interested reader is referred to the original papers, textbooks [18–20] and cited reviews for all technical details.

Confining ourselves by quenched approximation (exact in large N_c limit), we have $\langle \det K \rangle = \exp\langle \log \det K \rangle + \mathcal{O}(N_c^{-2})$ and hence the standard expression for Euclidean one-loop effective action:

$$\Gamma_{\text{eff}} = \frac{1}{V} \log Z = -2 \int_0^\infty \frac{dT}{T} \exp(-m^2 T) \cdot \langle Z[A, T] \rangle \quad (2.4)$$

where

$$Z[A, T] = \int \mathcal{D}x_\mu \int \mathcal{D}\psi_\mu \exp(-S_0) \text{tr P exp} \left(i \int_0^T d\tau (A_\mu \dot{x}_\mu - F_{\mu\nu} \psi_\mu \psi_\nu) \right) \quad (2.5)$$

with the free world-line action given by $S_0 = \int_0^T d\tau \left(\frac{1}{4} \dot{x}_\mu^2 + \frac{1}{2} \psi \dot{\psi} \right)$. We have included the factor $1/V$ in the integration measure $\mathcal{D}x_\mu$. The factor $2 = 4 \times (1/2)$ in (2.4) came from the trace over anticommuting coordinates (i.e. integration over the fermionic fields with the free action is normalized to unity in our conventions). All dynamical information is contained in the double average (over gauge fields and over quark trajectories) of the spinor Wilson loop

$$w(T) \equiv \langle Z[A, T] \rangle \quad (2.6)$$

There are several limiting cases where one can successfully study the behavior of $w(T)$ or related functions. Of prime importance is the nonrelativistic limit of large mass m , where the target space for the contours x_μ becomes effectively three-dimensional and introducing the einbein fields for dynamical masses one can systematically explore constituent picture of hadrons (see, e.g. [14, 21] and references therein). However for the trace (and hence

for the closed contours) we are discussing at the moment nonrelativistically suppressed backward-in-time trajectories are as important as forward-in-time ones and to address this problem in einbein fields formalism one probably has to use some alternative methods (see, e.g. [22]).

The second important case corresponds to the small T asymptotic. In a way, this is the standard operator product expansion [23]. In context of the theory of gravity it corresponds to well known Schwinger - DeWitt expansion [24]. One is to expand the Wilson loop in powers of fields in this limit. For constant background fields this is the way one obtains the effective Lagrangians of Heisenberg-Euler type. The typical term of this expansion looks like

$$\langle D^{k_1} F^{m_1} D^{k_2} F^{m_2} \dots D^{k_p} F^{m_p} \rangle \cdot T^l \tag{2.7}$$

The leading term represents the so called heavy quark condensate [9]

$$\langle \bar{q}q \rangle = -\frac{1}{12m} \left\langle \frac{\alpha_s}{\pi} F_{\mu\nu}^a F_{\mu\nu}^a \right\rangle \tag{2.8}$$

(see also [25], where the heavy quark condensate is discussed in the path integral formalism, including nonzero temperature case). It is clear that this expansion is unapplicable in the limit of vanishing mass m .

The phenomenon of CSB is related to the large proper time asymptotic of $w(T)$. As it was noticed by Banks and Casher in their seminal paper [5], for spontaneous CSB one should have

$$w(T) \sim \frac{c}{\sqrt{T}} \text{ at } T \rightarrow \infty \tag{2.9}$$

Indeed, it is easy to see from (2.3) and (2.4) that

$$\langle \bar{q}q \rangle = -\frac{\partial \Gamma_{\text{eff}}}{\partial m} \sim -4m \int_0^\infty dT \exp(-m^2 T) \frac{c}{\sqrt{T}} = -4\sqrt{\pi} \cdot c \tag{2.10}$$

Taking into account that in free case (i.e. without gauge fields) $w(T)$ is given by

$$w_0(T) = (4\pi T)^{-\frac{d}{2}} \tag{2.11}$$

one can say that the quark is dynamically forced to move effectively in 1 dimension instead of $3 + 1$ and this is the cause for CSB in this framework.

There are a few well known cases where the dimensional reduction of this kind indeed takes place. However, it is worth noticing that the condition (2.9) is rather restrictive to be incorporated in a simple way into the standard background field formalism. Indeed, one can show on general grounds (see [26 – 28] and [29] for review) that the typical large T asymptotic of the heat kernel trace $\text{Tr } \mathcal{K}(T) = \int dx \mathcal{K}(T, x, x)$, entering the effective action as

$$\Gamma_{1-loop} = \frac{1}{2} \int_0^\infty \frac{dT}{T} \text{Tr } \mathcal{K}(T) \tag{2.12}$$

in generic *fixed* background field is given by

$$\text{Tr } \mathcal{K}(T) = \frac{1}{(4\pi T)^{d/2}} \left(TW_0 + W_1 + \frac{1}{T} W_2 + \dots \right) \tag{2.13}$$

where nonlocal factors W_n can be expressed as integrals of the corresponding zero modes. Notice that the expansion goes in integer powers of $1/T$. We will address this contradiction between (2.13) and (2.9) in the section 4.

Simple illustrative example is Heisenberg - Euler effective Lagrangian [30] for constant magnetic field

$$L = -\frac{1}{8\pi^2} \int_0^\infty \frac{dT}{T} \exp(-m^2 T) \cdot \left(\frac{eH}{T} \operatorname{cth} eHT - \frac{1}{T^2} - \frac{1}{3}(eH)^2 \right) \quad (2.14)$$

The lowest energy level of the massless fermion from Dirac sea in constant magnetic field is zero, hence the absence of exponential damping in (2.14), $\operatorname{cth} eHT \rightarrow 1$ with $T \rightarrow \infty$. The factor $1/T = T \cdot T^{-4/2}$ in front of the $\operatorname{cth} eHT$ corresponds to the fact that in four-dimensional space-time there are two directions the fermion can move along as a free particle, while the dynamics in two other directions is confined by the field.³ This is nothing but the first term in the rhs of (2.13). On the other hand, we see something new here with respect to (2.13). The leading term at large T is the last term in the rhs of (2.14), which is $\mathcal{O}(T^0)$ and not $\mathcal{O}(T^{-1})$. This term represents one-loop short distance charge renormalization and, at the same time, the leading strong-field (i.e. small mass) logarithmic asymptotic of (2.14):

$$L = \frac{e^2 H^2}{24\pi^2} \log \frac{eH}{m^2} \quad (2.15)$$

This phenomenon of strong field - short distance duality [31] (small T - large T duality in our context) is quite general (see, e.g. recent discussion in [32]) and provides interesting possibilities for OPE subseries summation (see discussion below). Thus we see that quantum dynamics (the necessity to express the answer in terms of renormalized quantities in this case) can make the result (2.13) inapplicable.

3. Effective action at Gaussian level

The spinor Wilson loop factor $Z[A, T]$ given by (2.5) can be expanded in powers of fields and derivatives. It is convenient to use Fock-Schwinger gauge condition, which is a particular case of the so called generalized contour (or coordinate) gauge [33]. The latter is defined in terms of the oriented non-selfintersecting contour $z_\mu(s)$ as

$$A_\mu(z(s)) \frac{\partial z_\mu(s)}{\partial s} = 0 \quad (3.1)$$

The simplest contour gauge one usually uses is the Fock-Schwinger gauge condition with $z_\mu(s) = x_\mu^{(0)} + s(x - x^{(0)})_\mu$. In this gauge $\partial^2 z_\mu / \partial s^2 = 0$ and thanks to that one can express the vector-potential in the following form (compare with (B.3) from appendix B)

$$A_\mu^a(x) = \int_0^1 s ds y_\rho \exp(sy_\sigma D_\sigma)^{ab} F_{\rho\mu}^b(x^{(0)}) \quad (3.2)$$

where $y = x - x^{(0)}$ and Latin indices $a, b = 1, \dots, N^2 - 1$ stay for adjoint color.

³For the field $\mathbf{H} = \mathbf{e}_z H$, the particle moves freely in z -directions (along the field) and in t -direction (no force acts on the particle in rest).

It is worth stressing that for contour gauges gauge invariance corresponds to the contour independence, not just $x^{(0)}$ independence, as is sometimes posed. This situation is analogous to the covariant R_ξ -gauges, where ξ -independence is necessary but not sufficient condition of gauge invariance; in other words one can easily construct gauge-noninvariant (and hence physically unobservable) but ξ -independent quantity. The contour independence is restored only in the full Wilson loop, but not at any given order of the expansion over fields and/or derivatives (neither at any given order of the covariant perturbation theory [26, 27]). Practically it means that general expansion of some nonlocal object like the Wilson loop over local condensates has no universal coefficients, independent on the choice of contours used to fix the gauge (see the appendix B).

With these reservations in mind, we can proceed and expand the Wilson loop over gluon fields. In quantum case the dynamics is determined by the average over quark trajectories $y_\mu(\tau)$ (and over spinor “coordinates” $\psi_\mu(\tau)$) with Gaussian weight $\exp(-S_0)$ and over the vacuum gluon fields with the standard Yang-Mills action. Having performed the latter average, the first two nontrivial terms in the expansion of $Z[A, T]$ take the form

$$\begin{aligned} \langle Z[A, T] \rangle \approx & w_0(T) + \int \mathcal{D}y_\mu \int \mathcal{D}\psi_\mu \exp(-S_0) \int_0^T d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^1 ds_1 \int_0^1 ds_2 \\ & (s_1 \dot{y}_{\mu_1}(\tau_1) y_{\rho_1}(\tau_1) - \psi_{\mu_1}(\tau_1) \psi_{\rho_1}(\tau_1) \delta(1 - s_1)) \\ & \cdot (s_2 \dot{y}_{\mu_2}(\tau_2) y_{\rho_2}(\tau_2) - \psi_{\mu_2}(\tau_2) \psi_{\rho_2}(\tau_2) \delta(1 - s_2)) \cdot \\ & \cdot \left\langle \text{Tr} \exp(s_2 y(\tau_2) D) F_{\mu_2 \rho_2}(x^{(0)}) \exp(s_1 y(\tau_1) D) F_{\mu_1 \rho_1}(x^{(0)}) \right\rangle \end{aligned} \quad (3.3)$$

The computational technique for such integrals is well developed [15–17, 34]. The basic ingredients are the one-dimensional Green’s functions on a circle $G_B(\tau_1, \tau_2)$ and $G_F(\tau_1, \tau_2)$ defined by

$$\langle y_\mu(\tau_1) y_\nu(\tau_2) \rangle_y = -\delta_{\mu\nu} G_B(\tau_1, \tau_2) = \delta_{\mu\nu} \left(|\tau_1 - \tau_2| - \frac{(\tau_1 - \tau_2)^2}{T} \right) \quad (3.4)$$

and

$$\langle \psi_\mu(\tau_1) \psi_\nu(\tau_2) \rangle_\psi = \frac{\delta_{\mu\nu}}{2} G_F(\tau_1, \tau_2) = \frac{\delta_{\mu\nu}}{2} \text{sign}(\tau_1 - \tau_2) \quad (3.5)$$

The following identity is of special use

$$\left\langle \exp \left(y_\mu(\tau_1) k_\mu^{(1)} \right) \exp \left(y_\nu(\tau_2) k_\nu^{(2)} \right) \right\rangle_y = \exp \left(-G_B(\tau_1, \tau_2) k_\mu^{(1)} k_\mu^{(2)} \right) \quad (3.6)$$

The result is given by the expressions

$$\Gamma_{\text{eff}}^{(2)} = -2 \int_0^\infty \frac{dT}{T} \exp(-m^2 T) w_0(T) \cdot T^2 K(T) \quad (3.7)$$

where $w_0(T)$ is defined by (2.11), while $K(T)$ reads

$$K(T) = \langle \text{Tr} F_{\mu\nu}(x^{(0)}) \mathcal{F}(\xi) F_{\mu\nu}(x^{(0)}) \rangle \quad (3.8)$$

with the formfactor

$$\mathcal{F}(\xi) = \int_0^1 du u(1-u) \exp(u(1-u)\xi) \quad (3.9)$$

and $\xi = T\overrightarrow{D}_\sigma\overleftarrow{D}_\sigma$. The arrows indicate that the derivatives act on the right. The contraction of indices in (3.8) is worth noticing (see [35] in this respect). The most nontrivial thing is the exact expression for the formfactor (3.9), which was obtained for the first time in [26] for classical backgrounds (and effectively reproduced in [34] using Feynman-Schwinger technique). For large m eqs. (3.7)–(3.9) lead to the expression (2.8).

4. Large- T asymptotic limit and CSB

We are to study large- T asymptotic of (3.7). One has $\mathcal{F}(\xi) \sim 1/\xi^2$ as $\xi \rightarrow -\infty$. It is easy to show that for the scalar particle one would have $\mathcal{F}(\xi) \sim 1/\xi$ as $\xi \rightarrow -\infty$. This important difference corresponds to the fact that no exponential damping at large T other than $\exp(-m^2T)$ is possible for fermions in the chiral limit. Naively one has

$$\lim_{T \rightarrow \infty} T^2 K(T) = 2 \langle \text{Tr} F_{\mu\nu}(x^{(0)}) D^{-4} F_{\mu\nu}(x^{(0)}) \rangle \quad (4.1)$$

This expression is formal, however, due to infrared divergencies. We will show that general asymptotic expansion at large T may contain logarithmic terms of the form

$$T^2 K(T) = c_0 \log\left(\frac{T}{\lambda^2}\right) + \mathcal{O}(T^{-1} \log T) \quad \text{for } T \rightarrow \infty \quad (4.2)$$

where λ is typical correlation length and c_0 - some dimensionless coefficient. Possible subleading T -independent contribution in the left hand side of (4.2) is included into the definition of λ .

Let us show how the behavior (4.2) of $K(T)$ follows. The asymptotic pattern is controlled by the function

$$f(s) = \langle \text{Tr} F_{\mu\nu}(x^{(0)}) \exp\left(s\overrightarrow{D}_\sigma\overleftarrow{D}_\sigma\right) F_{\mu\nu}(x^{(0)}) \rangle \quad (4.3)$$

which one needs to know both at large and at small s . The small proper time asymptotic of $f(s)$ is given by the standard OPE:

$$f(s) = \langle \text{Tr} F_{\mu\nu}^2 \rangle + s \langle \text{Tr} F_{\mu\nu} D^2 F_{\mu\nu} \rangle + \mathcal{O}(s^2) \quad (4.4)$$

As usual in SVZ sum rules, it is assumed by definition that all perturbative contributions are subtracted from each term of (4.4), thus defining the genuine nonperturbative function (4.3). In the framework of Wilson OPE each term in (4.4) depends on the dynamical scale μ (separating contributions of perturbative coefficient functions and nonperturbative matrix elements). The subtle question about μ - dependence of the function $f(s)$ and the effective action is somewhat beyond our main line and will not be discussed. Taking more phenomenological attitude, the reader may think of $f(s)$ as being computed on nonperturbative field configurations of one's favorite QCD vacuum ensemble (instantons, dyons, P-vortices etc); this would correspond to some effective $\mu \approx 1 \text{ GeV}$ of the order of the onset of nonperturbative dynamics.

The Gaussian approximation to the function $f(s)$ defined by (4.3) can be written as (see details in appendix B)

$$f(s) \rightarrow f_2(s) = \frac{1}{(4\pi s)^2} \int d^4 l \exp(-l^2/4s) \langle \text{Tr} F_{\mu\nu}(x^{(0)}) \exp(l_\sigma \vec{D}_\sigma) F_{\mu\nu}(x^{(0)}) \rangle \quad (4.5)$$

The correlator in the right hand side (compare with (3.2)) is frequently used in the Gaussian stochastic scenario of confinement ([36], see review [21] and references therein). At distances l larger than some typical correlation length $\tilde{\lambda}$ this correlator decays exponentially⁴ with l . Therefore for $s\tilde{\lambda}^{-2} \gg 1$ the leading asymptotic is given by $f(s) \sim s^{-2}$. To be more precise, it is convenient to parameterize two-point correlator in the standard way [36]

$$\begin{aligned} \langle \text{Tr} F_{\mu\nu}(x^{(0)}) \exp(l_\alpha \vec{D}_\alpha) F_{\rho\sigma}(x^{(0)}) \rangle = & (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(l^2) + \\ & + \partial_\mu [(l_\rho\delta_{\nu\sigma} - l_\sigma\delta_{\nu\rho})D_1(l^2)] \\ & - \partial_\nu [(l_\rho\delta_{\mu\sigma} - l_\sigma\delta_{\mu\rho})D_1(l^2)] \end{aligned} \quad (4.6)$$

where both functions $D(l^2)$ and $D_1(l^2)$ exponentially decay at large distances. Correspondingly, one has

$$f_2(s) = f_D(s) + f_{D_1}(s) \quad (4.7)$$

with the following asymptotic limits at large s :

$$f_D(s) = \frac{\eta}{s^2} + \mathcal{O}(s^{-3}); \quad f_{D_1}(s) = \frac{\zeta}{s^3} + \mathcal{O}(s^{-4}) \quad (4.8)$$

The nonperturbative constants η, ζ are given in this approximation by

$$\eta = \frac{3}{4} \int_0^\infty dl^2 l^2 D(l^2); \quad \zeta = \frac{3}{16} \int_0^\infty dl^2 l^4 D_1(l^2) \quad (4.9)$$

The difference between asymptotic limits of $f_D(s)$ and $f_{D_1}(s)$ is of crucial importance. Indeed, combining (3.8), (3.9) and (4.8) one gets

$$K(T) \equiv K_D(T) + K_{D_1}(T) = \int_0^1 du u(1-u) [f_D(u(1-u)T) + f_{D_1}(u(1-u)T)] \quad (4.10)$$

and for $T \rightarrow \infty$ we have

$$T^2 K_D(T) \sim \log T, \quad \text{while } T^2 K_{D_1}(T) \sim \frac{\log T}{T} \quad (4.11)$$

The latter term being exponentiated cannot produce $T^{-1/2}$ term and hence the function $D_1(l^2)$ alone gives no spontaneous CSB. In other words, $D(l^2) \equiv 0$ implies unbroken chiral symmetry. On the other hand, it is well known [36] that just nonzero $D(l^2)$ is responsible for confinement, while $D_1(l^2)$ is not.

It is reasonable to expect that the pattern we have discussed at the level of two-point correlator is general. From higher correlators one would have $\sim \log T$ terms, which add

⁴Notice that this length $\tilde{\lambda}$ characterizes the function $D(l^2)$ and, generally speaking, $\tilde{\lambda} \neq \lambda$. On the other hand, it is physically natural to assume that $\tilde{\lambda}$ and λ are of the same order of magnitude.

up to the coefficient η , and $\sim \log^n T$ terms coming from the corresponding reducible part (i.e. the product of lower order correlators). The exponentiation of the series produces the desired power-like behavior of $w(T)$. All terms $\sim T^{-k} \log T$ are subleading and do not change the leading power.

Of course, we cannot compute from the first principles the coefficient in front of a general $\log^n T$ term of this series to make the above arguments quantitative. But confining ourself by Gaussian approximation we can proceed further. The actual results for the chiral condensate and other quantities will crucially depend on the profile of the function $f_2(s)$, which, in its turn, is determined by the functions $D(l^2)$ and $D_1(l^2)$. Namely, from (4.3) and (4.6) we have

$$\begin{aligned} T^2 K_D(T) &= \frac{3}{4} \int_0^\infty dl^2 l^2 D(l^2) \int_0^1 \frac{du}{u(1-u)} \exp\left(-\frac{l^2}{4Tu(1-u)}\right) \\ &= \frac{3}{2} \int_0^\infty dl^2 l^2 D(l^2) \exp\left(-\frac{l^2}{2T}\right) K_0\left(\frac{l^2}{2T}\right) = \\ &= \frac{3}{2} \int_0^\infty dl^2 l^2 D(l^2) \left[\log\left(\frac{4T}{e\gamma l^2}\right) + \frac{l^2}{2T} \log\left(\frac{4T}{e\gamma l^2}\right) + \mathcal{O}(T^{-2} \log T) \right] \end{aligned} \quad (4.12)$$

Thus the leading large- T asymptotic has the form

$$T^2 K_D(T) \rightarrow 2\eta \log\left(\frac{T}{\lambda^2}\right) \quad (4.13)$$

with the correlation length λ defined by

$$\log \lambda^2 = \frac{\int_0^\infty dl^2 l^2 D(l^2) \log(e\gamma l^2/4)}{\int_0^\infty dl^2 l^2 D(l^2)} \quad (4.14)$$

It is worth repeating that this leading logarithmic term is absent in the deconfinement phase where $D(l^2) = 0$.

To summarize, the leading large- T asymptotics (4.2) of Γ_{eff} in confining background from the Gaussian term reads

$$\Gamma^{(0)} + \Gamma_{\text{eff}}^{(2)} = -2 \int_0^\infty \frac{dT}{T} \exp(-m^2 T) w_0(T) \left(1 + 2\eta \log\left(\frac{T}{\lambda^2}\right) + \dots \right) \quad (4.15)$$

where both parameters η and λ are of essentially nonperturbative origin.

It is again instructive to compare (4.15) with the constant field case. For constant field the (unrenormalized) result (2.14) can be written as

$$\Gamma = -2 \int_0^\infty \frac{dT}{T} \exp(-m^2 T) w_0(T) \exp\left(\sum_{n=1}^\infty \kappa_n (eHT)^{2n}\right) \quad (4.16)$$

where

$$\kappa_n = 2 \frac{(-1)^{n+1} \zeta(2n)}{n \pi^{2n}} (2^{2n-1} - 1) \quad (4.17)$$

The ‘‘correlation length’’ for the constant field is infinite and in this sense T is never large, the series in n cannot be truncated and at the Gaussian level $K(T) \rightarrow \text{const} \neq 0$ for

$T \rightarrow \infty$. For finite correlation length typical term in the corresponding expansion does not increase as a power of T and $K(T) \sim \log T/T^2 \rightarrow 0$ as $T \rightarrow \infty$. In terms of (4.16) the duality [31] mentioned above corresponds to the fact that the *leading* large T asymptotic of renormalized effective action (i.e. the expression (2.15)) is controlled by the *lowest* term of the *unrenormalized* expression (4.16) (i.e. the coefficient κ_1).

The leading Gaussian large T behavior in confining background given by (4.15) is much softer than constant field answer (4.16). It is clear that the logarithmic term (4.2) as it is cannot lead to CSB (since we need power-like rise to get (2.9)). But partial summation of such terms can do the job. The most crucial point is the structure of the series in the right hand side of (4.15). No universal closed form expressions analogous to (3.8), (3.9) for all terms of higher orders are known (see, however, [28] for explicit form-factors of the third order). On the other hand, one can argue on physical grounds that terms

$$\frac{1}{n!} \left(2\eta \log \left(\frac{T}{\lambda^2} \right) \right)^n \tag{4.18}$$

(together with other ones) should present. Such terms correspond to the factorized part contribution of the higher order averages $\langle \text{Tr } FF \dots F \rangle$. The important role played by these factorized averages is known under the name of vacuum dominance for a long time and it is successfully used in sum rules approach. From general analysis of [5] we expect that it is the confinement property that causes the CSB and it is known for a long time that just described reduction of the gluon ensemble (known as Gaussian approximation in the context) provides confinement (see review [21] and references therein). Therefore one can hope that we have summed “many enough” terms to keep CSB. On the other hand, the connected parts we have omitted physically correspond to the exchanges by multi-gluon glueballs and gluelumps, which are heavy objects and hence their contribution is to be suppressed for the low energy physics (see [37] in this respect). Another argument comes from the abelian dominance picture (see review [38] and references therein). Since in the maximal abelian gauge the higher irreducible correlators are suppressed (because they correspond to diagonal — off-diagonal gluon couplings), one gets the same factorization pattern. It is important that the Gaussian factorization is to be assumed for the integration over x_μ as well (i.e. we speak about some kind of “rainbow” approximation and no contraction of $y^{(i)}, y^{(j)}$ belonging to different clusters is done).

Summing of this “Gaussian” subseries of the full Wilson loop average would result in the effective action

$$\Gamma_{\text{eff}} = -2 \int_0^\infty \frac{dT}{T} \exp(-m^2 T) w_0(T) \cdot [\exp(T^2 K(T)) + \dots] \tag{4.19}$$

where dots stay for the contributions of non-Gaussian terms. The spontaneous chiral symmetry breaking condition has the following form in considered Gaussian approximation:

$$\lim_{T \rightarrow \infty} T \frac{d}{dT} T^2 K(T) = \frac{d-1}{2} \tag{4.20}$$

or, in terms of (4.9)

$$\int_0^\infty dl^2 l^2 D(l^2) = 1 \tag{4.21}$$

in four dimensional space-time. Then for the condensate one gets

$$\langle \bar{q}q \rangle = -\frac{1}{4} \left(\frac{1}{\sqrt{\pi\lambda}} \right)^3 \tag{4.22}$$

where λ is defined by (4.14). The condensate vanishes in the deconfinement phase transition point.

The above result deserves a few comments. Physically, the parameter η is given by some integral moment of the function $D(l^2)$ and, at the first look can be of arbitrary value, while we need strictly $\eta = 3/4$ to get CSB. In a sense, this is an artefact of Gaussian approximation. It is worth repeating that higher terms in the r.h.s. of (4.15) bring both $\sim \log T$ terms, shifting pure Gaussian value of η , given by (4.9) to its pure “geometrical” value (4.20) and $\sim \log^n T$ terms, adding up to the exponent. To some extent it resembles well-known effect of unphysical surface dependence of the Wilson loop average computed in Gaussian approximation: to get rid of it one has to sum the full cluster expansion.⁵ Moreover, suppose that the Gaussian asymptotic law for (4.19) would be different from $\log T$ law (4.13). The conclusion in this case would be that Gaussian approximation does not provide CSB and one needs the full series (or some subseries other than Gaussian) in (3.3) to get (2.9). The actual conclusion is different: Gaussian reduction of the confining vacuum can lead to CSB (because of (4.13)) if it is done in a self-consistent (in the sense of (4.20), (4.21)) way.

On the other hand, all dimensionfull quantities in nonperturbative theory (like condensates, string tension, correlation lengths etc) are proportional to the corresponding power of Λ_{QCD} with some dimensionless coefficient, unequivocally fixed by the theory. Neither the ratio of such coefficients is a freely adjustable parameter. Since we discuss in this paper not the full theory in the gluon sector but rather its Gaussian reduction (still keeping confinement), the relation (4.21) can be understood as a kind of self-consistency condition for Gaussian approximation (if we wish it to provide CSB). The fact that such self-consistent reduction should introduce relations between condensates and correlation lengths is well known [39]. The ultimate reason for that are the Bianchi identities: correlator of derivatives $\epsilon_{\rho\mu\nu\sigma} \partial_\rho F_{\mu\nu}(x, x^{(0)})$ with any operator, for example, $F_{\alpha\beta}(y, x^{(0)})$ (inversely proportional to some typical correlation length) is given by the higher correlators of F 's (another term, containing Bianchi form, vanishes). But the higher correlators factorize to the product of Gaussian ones in the chosen approximation.

It is clear from the discussion that we were mostly interested in qualitative pattern of the effect. Nevertheless it may be interesting to compare the results with the existing phenomenology of Gaussian stochastic picture of QCD vacuum. Standard way of representing the lattice data on the nonperturbative function $D(l^2)$ is

$$D(l^2) = D(0)P\left(l/\tilde{\lambda}\right) \exp\left(-l/\tilde{\lambda}\right) \tag{4.23}$$

where $P(x)$ is some rational polynomial, normalized by condition $P(0) = 1$. Lattice [11] data correspond to $\tilde{\lambda} = 0.22$ Fm for quenched SU(3) case, while the value of dimensionless

⁵This analogy should not be understood literally, of course, there is no any special surface in the discussed problem, since we sum over all trajectories of the light quark.

product $D(0)\tilde{\lambda}^4$ is numerically about (0.15 ± 0.05) . Unfortunately, the poor knowledge of the pre-exponential factor $P(x)$ precludes one to make quantitative predictions from the expressions (4.21), (4.22). However, it is interesting to notice that the simplest ansatz $P(x) \equiv 1$ does not describe data satisfactory;⁶ it predicts $D(0)\tilde{\lambda}^4 = 1/12$ and too small value of the chiral condensate $\langle \bar{q}q \rangle = -(140 \text{ MeV})^3$ instead of the correct value $\langle \bar{q}q \rangle = -(250 \text{ MeV})^3$. Of course, one can easily fit $P(x)$ to get the desired numbers. But this seems to be misleading since, as it has been already mentioned, eventually these are the higher order terms which shift η and $\langle \bar{q}q \rangle$ to their correct values.

5. Conclusion

We have studied the chiral symmetry breaking at one-loop level in the background of confining gluon fields. The latter is taken in the Gaussian picture, i.e. we have omitted higher than two-point irreducible condensates. This approximation is supported by the sum-rule phenomenology as well as by more sophisticated analytical and lattice analysis. It is worth stressing that the vacuum gluon fields ensemble reduced in such way still has the confinement property. It is shown that this vacuum breaks chiral symmetry spontaneously provided the large proper-time asymptotic of the operator product expansion has the form (4.20).

At the same time it is worth stressing that despite “Gaussian reduction” alone is enough to get CSB, it does not mean that higher order terms are not important. There are at least two respects where they are: first, their presence is a matter of principle for correctness of the general Wilson loop asymptotic (see, e.g. comments in [21]) like surface independence etc; and second, they can be numerically significant (say, contribute 10-15%) in the quantities like η , ζ .

We have also discussed that perturbative asymptotic of the corresponding correlators cannot provide CSB; roughly speaking, at small momenta perturbative Green’s functions are “too soft”. No any specific topological properties of the vacuum gluon fields show up in our analysis, the only relevant thing is the large proper time asymptotic of the OPE (resulting from confinement). We did not use in any place explicit expressions for the gluon fields profile, only the vacuum averages entered our analysis. Nevertheless it would be interesting to understand the relation of the discussed issues with the well established phenomenology of CSB in, e.g. instanton backgrounds (without confinement *per se*), having in mind that it is ultimately the strong confining color forces that determine the dynamics of CSB in accordance with general analysis of [8].

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⁶Notice that this ansatz approaches the point $l = 0$ linearly: $D(l^2) - D(0) \sim \sqrt{l^2}$ instead of the behavior $D(l^2) - D(0) \sim l^2$ dictated by (4.5)

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Note added. The related issues are discussed in the recent preprint hep-th/0511176 by V.Vyas we got after our paper had been completed. The main interest of the cited paper concentrates on the consequences the asymptotic law (2.9) has for the behavior of two-point correlators of hadronic currents, while we have tried to answer another and physically different question: how can the asymptotic (2.9) itself be obtained using the OPE language.

A. Feynman-Schwinger method: the basic formulas

Throughout the paper we use Feynman-Schwinger world-line integration technique. The standard building blocks here are the following:

1. γ_5 - invariance of the determinant (we define $D_\mu = \partial_\mu - iA_\mu$):

$$\begin{aligned} \log \det (\gamma_\mu D_\mu + m) &= \frac{1}{2} \log \det ([\gamma_\mu D_\mu + m] [-\gamma_\mu D_\mu + m]) \\ &= \frac{1}{2} \log \det \left(-D^2 + \frac{ig}{4} F_{\mu\nu} [\gamma_\mu \gamma_\nu] + m^2 \right) \end{aligned} \quad (\text{A.1})$$

2. proper-time representation of the logarithm:

$$\log A = - \int_0^\infty \frac{dT}{T} \exp(-AT) \quad (\text{A.2})$$

The above expression is of course symbolical and properly regularized form of (A.2) must be used in actual computations. The typical examples are given by exact equality

$$\log A + \gamma = - \lim_{\xi \rightarrow 0} \int_0^\infty dT T^{\xi-1} (1 + \xi \log T) \exp(-AT) \quad (\text{A.3})$$

which can be obtained differentiating the identity

$$A^{-\xi} \Gamma(1 + \xi) = \xi \int_0^\infty dT T^{\xi-1} \exp(-AT)$$

or by frequently used integral

$$\log A = - \int_0^\infty \frac{dT}{T} (\exp(-AT) - \exp(-T)) \quad (\text{A.4})$$

3. integration over commuting paths replacing the bosonic traces:

$$\begin{aligned} \text{Tr} \exp(-T(-D^2)) &= \int \frac{d^4 p}{(2\pi)^4} \langle p | \exp(-(p + gA)^2 T) | p \rangle \\ &= \int \mathcal{D}x \exp\left(-\frac{1}{4} \int_0^T \dot{x}_\mu^2 d\tau\right) \text{tr P exp}\left(ig \int_0^T A_\mu(x) \dot{x}_\mu d\tau\right) \end{aligned} \quad (\text{A.5})$$

4. integration over anti-commuting fields representing the path-ordered exponent of the gamma matrices:

$$\text{Tr P exp} \left(-\frac{ig}{4} \int_0^T d\tau F_{\mu\nu} [\gamma_\mu \gamma_\nu] \right) = \int \mathcal{D}\psi \text{tr P exp} \left(-\int_0^T d\tau \left(\frac{1}{2} \psi_\mu \dot{\psi}_\mu - ig F_{\mu\nu} \psi_\mu \psi_\nu \right) \right) \quad (\text{A.6})$$

where the small trace tr goes over color indices only. The corresponding operators $\hat{\psi}$ anti-commute and act on the Hilbert space of Dirac spinors $|\alpha\rangle$ as

$$\psi_\mu \psi_\nu + \psi_\nu \psi_\mu = \delta_{\mu\nu}; \quad \psi^\mu |\alpha\rangle = \frac{1}{\sqrt{2}} \gamma_{\alpha\beta}^\mu |\beta\rangle \quad (\text{A.7})$$

The normalization is provided by

$$\int \mathcal{D}\psi \exp \left(-\frac{1}{2} \int_0^T d\tau \psi_\mu \dot{\psi}_\mu \right) = 1 \quad (\text{A.8})$$

The integration goes over periodic commuting $x_\mu(\tau)$ and anti-commuting $\psi_\mu(\tau)$, defined on the circle of the length T .

B. On the contour dependence of nonlocal quantities

Let us consider the gauge invariant phase factor along small closed contour C for a particular inhomogeneous field A_μ

$$W(C) = \Phi(x, x) = \text{Tr P exp} \left(i \oint_C A_\mu dz_\mu \right) \quad (\text{B.1})$$

The naive expansion over fields has the following form:

$$W(C) \approx 1 + i^2 \oint_C dz_\mu \oint_C^z du_\nu \text{Tr } A_\nu(u) A_\mu(z) + \dots \quad (\text{B.2})$$

The second term is gauge non-invariant (in nonabelian case). If one rewrites it in contour gauge [33] where

$$A_\mu(x) = \int_0^1 ds \frac{\partial z_\alpha(s)}{\partial s} \frac{\partial z_\beta(s)}{\partial x_\mu} F_{\alpha\beta}(z(s)) \quad (\text{B.3})$$

the answer would be contour dependent. One can rearrange the series (B.2) to make it manifestly gauge-invariant:

$$W(C) \approx 1 + i^2 \int_S d\sigma_{\mu\rho}(z) \int_{S_z} d\sigma_{\nu\sigma}(u) \text{Tr } F_{\mu\rho}(z, x^{(0)}) F_{\nu\sigma}(u, x^{(0)}) \quad (\text{B.4})$$

where $x^{(0)} = z(s=0)$ and the shifted field strength is given by

$$F_{\mu\rho}(z, x^{(0)}) = \Phi(x^{(0)}, z) F_{\mu\rho}(z) \Phi(z, x^{(0)}) \quad (\text{B.5})$$

This is nothing but the nonabelian Stokes theorem [33, 40]. Notice that despite the Wilson loop itself is gauge-invariant and $x^{(0)}$ -independent (it depends only on the contour C but

not on the integration surface S in (B.4)), the second term in the right hand side of (B.4) is gauge-invariant *but* contour dependent. Its SVZ-like expansion over condensate and derivatives would manifestly depend on the particular choice of contours $z_\mu(s)$ and will be *different* for different contour gauges (i.e. for different choices of $z_\alpha(s)$ and hence S).

However, in Gaussian approximation the account for contour dependence would not be legitimate [36]. The reason is that the variation of the nonlocal Gaussian correlator

$$\langle \text{Tr} F_{\mu\rho}(z, x^{(0)}) F_{\nu\sigma}(w, x^{(0)}) \rangle$$

over contours brings additional powers of $F_{\alpha\beta}$, and hence it is proportional to the correlators of higher orders (see related discussion in [21]). We have used the same phenomenon replacing (4.3) by (4.5) in the main text. Indeed, let us look at the power expansions of those functions. They are given by

$$f(s) = \langle \text{Tr} F_{\mu\nu} \left(1 + sD_\alpha^2 + \frac{s^2}{2}(D_\alpha^2)^2 + \dots \right) F_{\mu\nu} \rangle \quad (\text{B.6})$$

and

$$f_2(s) = \langle \text{Tr} F_{\mu\nu} \left(1 + sD_\alpha^2 + \frac{s^2}{2} \left[(D_\alpha^2)^2 - \frac{2}{3} D_\alpha i F_{\alpha\beta} D_\beta + \frac{1}{6} F_{\alpha\beta}^2 \right] + \dots \right) F_{\mu\nu} \rangle \quad (\text{B.7})$$

It is seen that the difference between $f(s)$ and $f_2(s)$ contains correlators of higher orders, resulting from non-commutativity of D_μ 's. In Gaussian approximation each power of derivative corresponds to large factor $\tilde{\lambda}^{-1}$ and in this sense to extract the leading term one can always neglect the commutators $[D_\alpha D_\beta]$. The validity of Gaussian approximation, in its turn, is controlled by the dimensionless parameter

$$\kappa = \langle \text{Tr} F_{\mu\nu}^2 \rangle \tilde{\lambda}^4$$

which is assumed to be small (it was mentioned above that, for example, $D(0)\tilde{\lambda}^4$ is about 1/10 according to the lattice data). In physical terms, Gaussian vacuum is short-correlated, and the nonperturbative gluon condensate is small in units of the lightest gluelump mass (the latter is of the order of $1/\tilde{\lambda}$). Using the language of covariant perturbation theory one can say that we have summed up only terms of the kind $\langle \text{Tr} F(D^2)^n F \rangle$ and not the terms with lesser number of derivatives.

To summarize, we have

$$f(s) = f_2(s) + \mathcal{O}(\kappa^{1+r})$$

where r is some positive number. Therefore the leading Gaussian asymptotic of $f(s)$ is given by the asymptotic of $f_2(s)$ found in eq. (4.8) in the main text.

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